Examples of relative Trisections

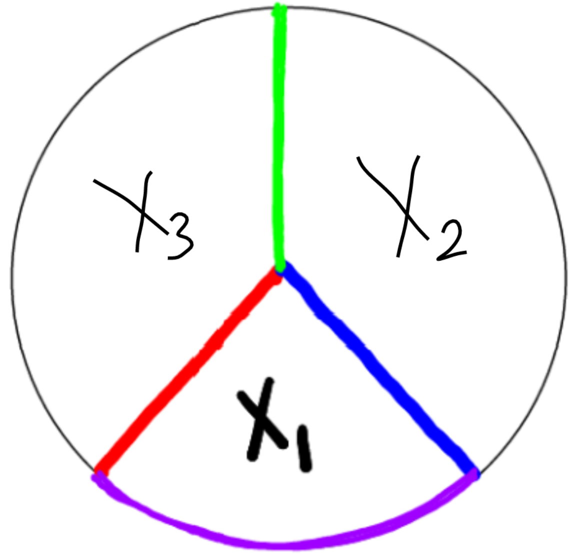
I. Definitions

II. Fillings of open books

III. Pieces in cut-and-paste operations

Relative Trisections

Def (A modification of Gay - Kirby) a relative trisection of a 4-mfd X with $\partial X \neq \phi$, ∂X connected is a decomposition $X = X_1 \cup X_2 \cup X_3$



- $X_i \cong \natural^k S^1 \times B^3$
- $\partial X_i \cong \#^k S^1 \times S^2$
- $X_i \cap X_{i-1}$ and $X_i \cap X_{i+1}$ $\cong$ 3-dim handlebody
- $X_i \cap \partial X \cong I \times P \cup \left(\begin{array}{c}\text{a third}\\ \text{of a disk}\end{array}\right) \times \partial P$, P a genus p surface with b bdry components
- $X_1 \cap X_2 \cap X_3 = F$ a genus g surface with b bdry components

Thm (Gay - Kirby) Let X be a smooth 4-mfd with non-empty and connected ∂X. For every OBD of ∂X, there exists a relative trisection of X.

"Open books can be filled with trisections".

Recall: An (abstract) open book decomposition is a pair (P, μ) where

- P is a surface with boundary
- $\mu : P \longrightarrow P$ diffeo, $\mu|_{\partial P} = id$.

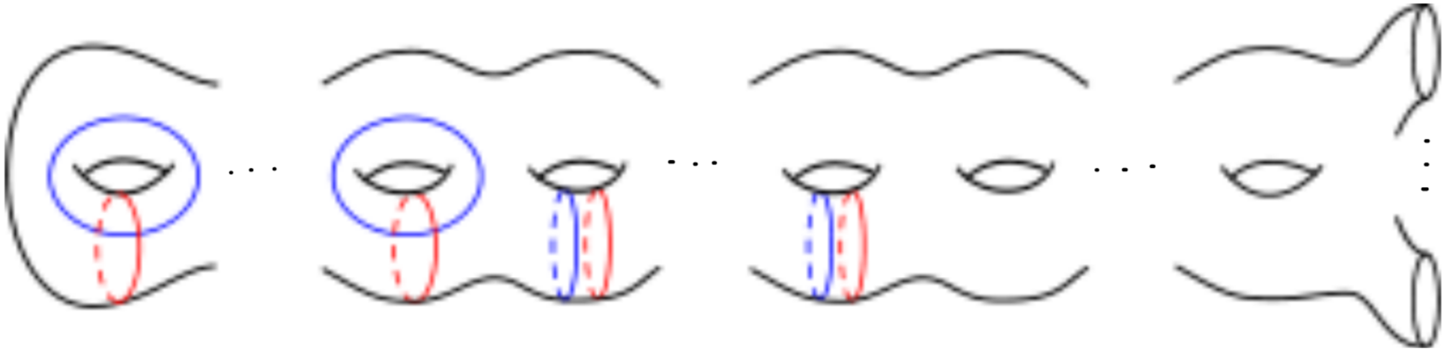
A 3-mfd Y admits an open book decomposition if

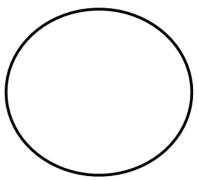
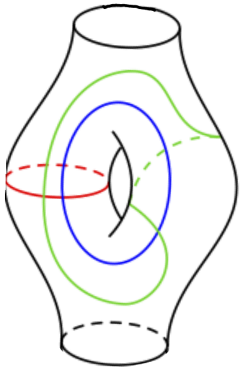
$$Y \cong \left(I \times P \big/ (0,x) \sim (1, \mu(x)) \right) \cup D^2 \times \partial P$$

Diagrams of relative trisections

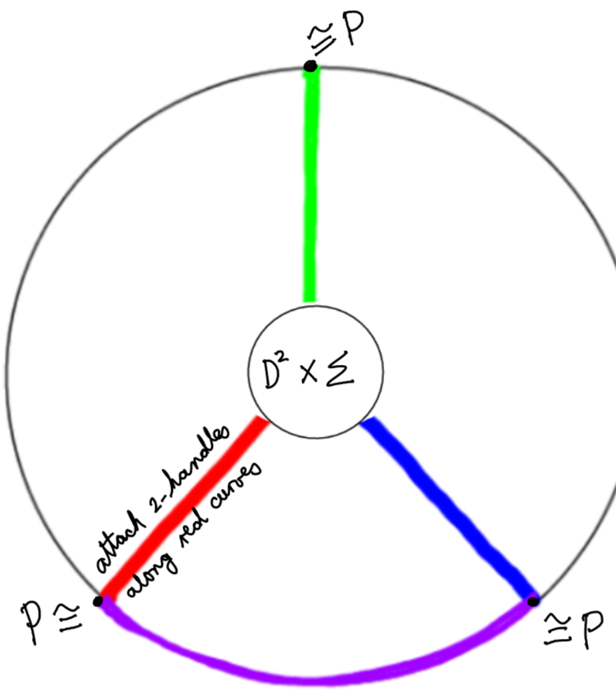
Def (Castro - Gay - P.) A relative trisection diagram is a tuple $(\Sigma, \alpha, \beta, \gamma)$ such that

$$\left.\begin{array}{l}(\Sigma, \alpha, \beta) \\ (\Sigma, \beta, \gamma) \\ (\Sigma, \gamma, \alpha)\end{array}\right\}$$ are diffeomorphism and handle slide equivalent to



Examples:  , 

Diagrams of Relative Trisections



A diagram gives instructions to build a 4-mfd

(0) start with $D^2 \times \Sigma$

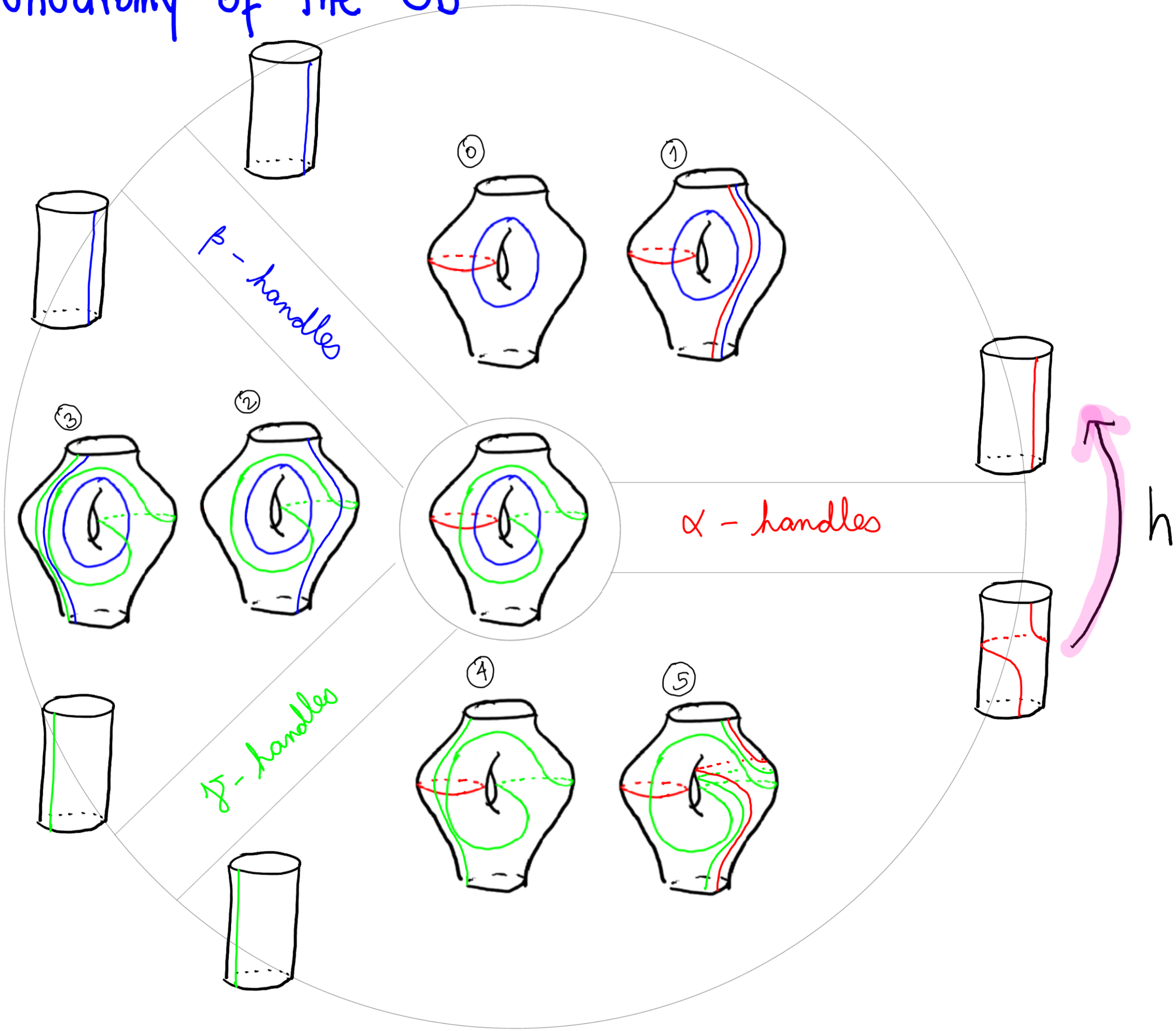
(1) select 3 different points $x_1, x_2, x_3 \in \partial D^2$ attach 2-handles to $\{x_i\} \times \Sigma$ along curves

(2) Glue in $I \times P \cup$ (third of disk) $\times \partial P$ to complete, $\#^k S^1 \times S^2$

(3) Fill in $\#^k S^1 \times S^2$ with $\natural^k S^1 \times B^3$

Notice: (1) and the definition tell us that the page P can be obtained from Σ by doing surgery along the curves in α, β of $\mathcal{J}$

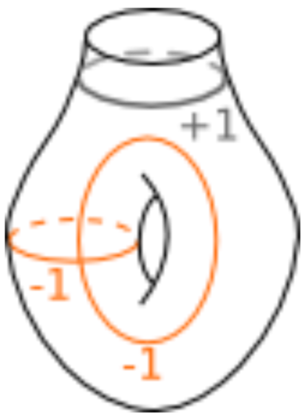
Finding the monodromy of the OB



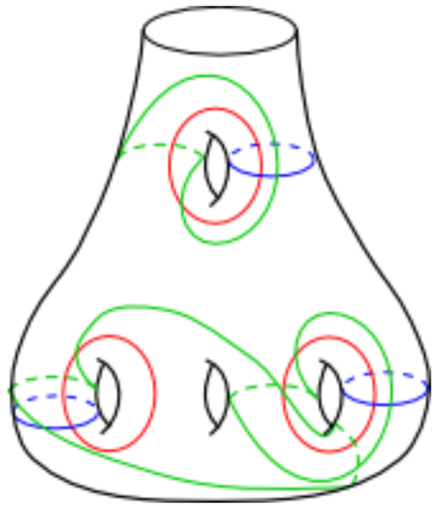
Fillings of open books

1. $\mathbb{P}$ as -1 surgery along $T_{2,-3}$

Surgery on open book of S^3 given by $T_{2,-3}$ plumbing



Trisection of $B^4 \cup$ 2-handle

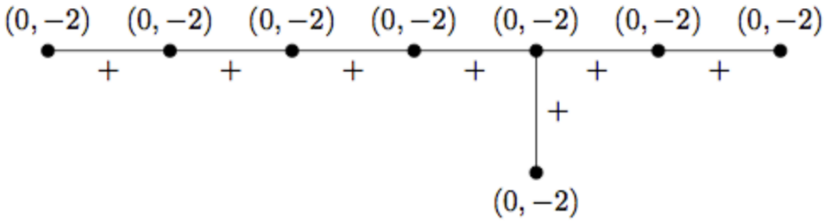


Right-veering monodromy $\Rightarrow$ open book supports a tight contact structure

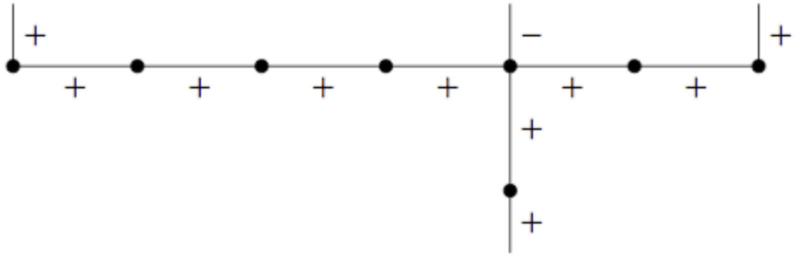
$b_2(B^4 \cup \text{2-handle}) = 1 < 8 \Rightarrow$ $B^4 \cup$ 2-handle is not homeomorphic to E_8 manifold $\Rightarrow$ $B^4 \cup$ 2-handle is not Stein

Fillings of open books

2. P as boundary of E_8 with planar open book

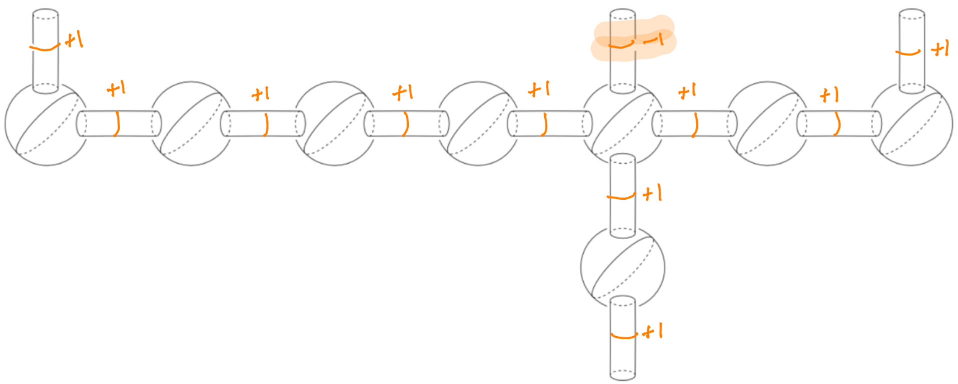


(a) The plumbing graph E_8.



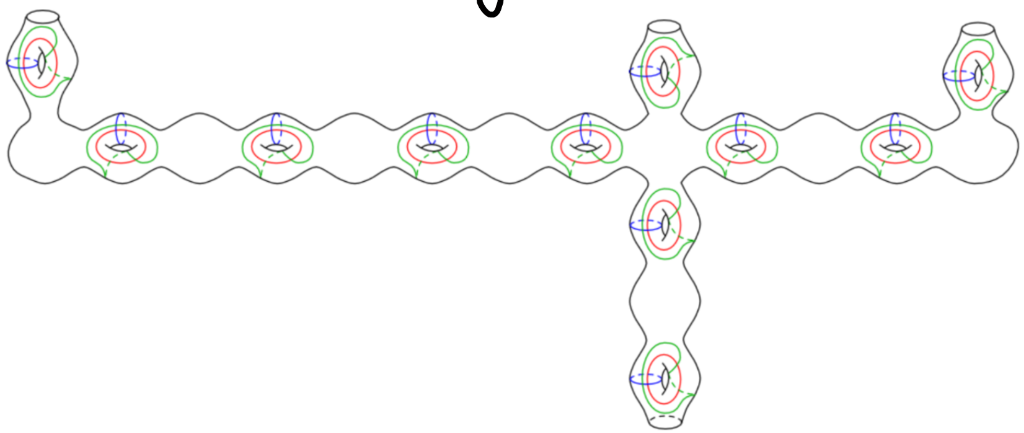
(b) The modified plumbing graph E_8^*

Planar open book



Monodromy not right-veering ⇒ contact structure not tight

Trisection diagram

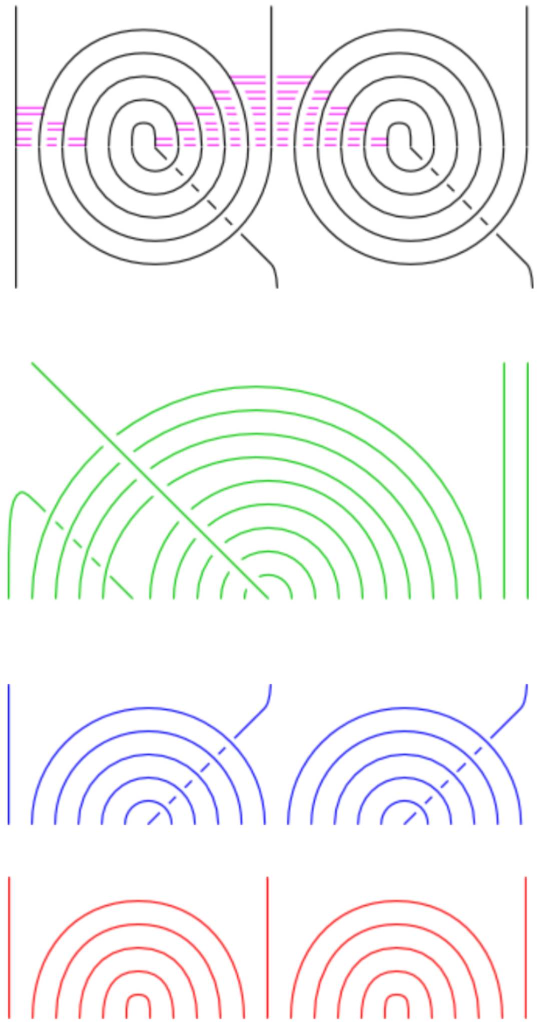


Lefschetz fibration not positive ⇒ not Stein structure

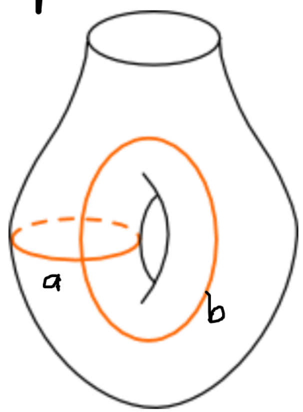
Fillings of open books

3. $\mathbb{P}$ as 2-fold cover of S^3 branched over $T_{3,5}$

Braided surface



Open book

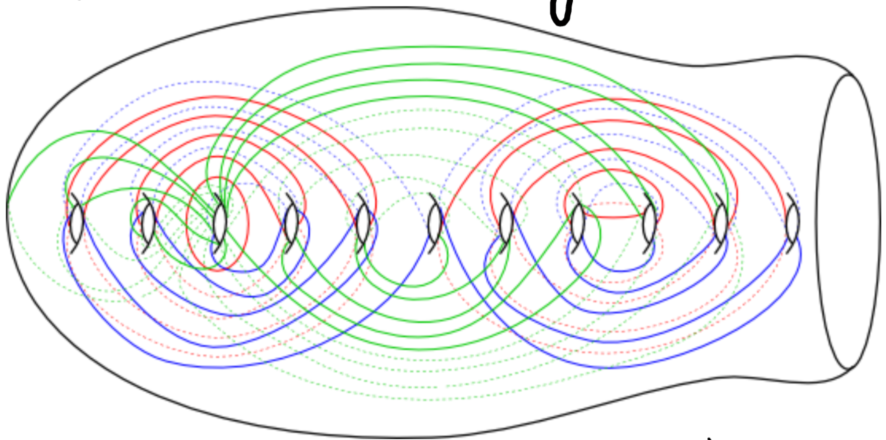


Monodromy is $(\tau_a \tau_b)^5$

$\Rightarrow$ positive

$\Rightarrow$ tight

Trisection diagram



Branched cover over positive surface

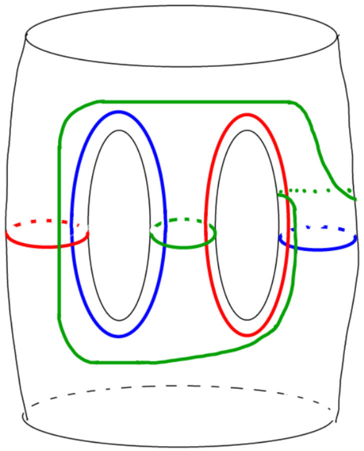
$\Rightarrow$ Stein

Pieces of constructions of exotica

1. Gluck twists

$$X = X \setminus N(S^2) \cup S^2 \times D^2$$

$$X' = X \setminus N(S^2) \cup_{\varphi} S^2 \times D^2$$

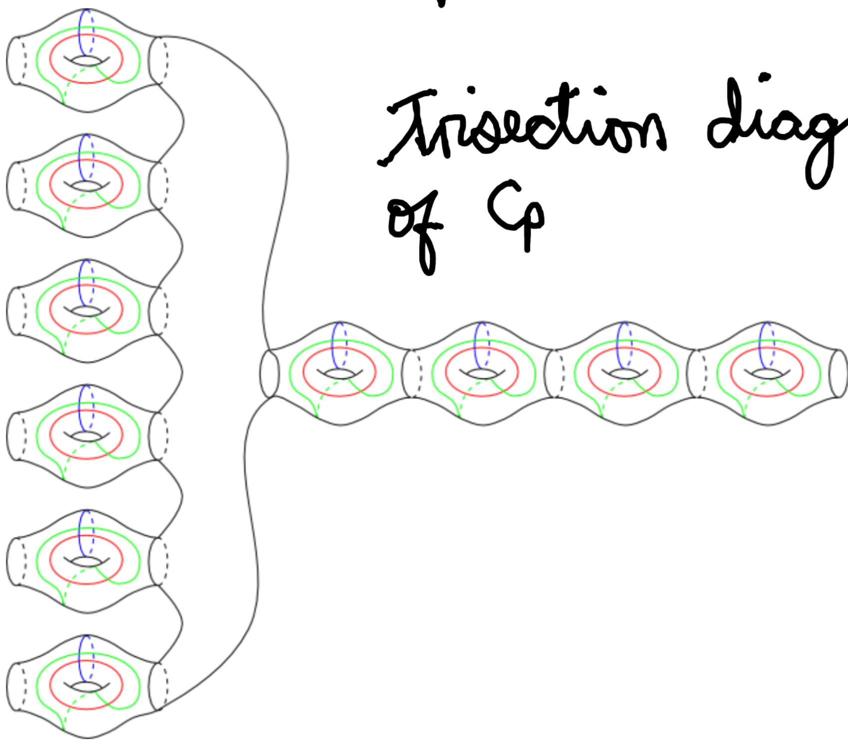


Trisection diagram for a D^2 bundle over S^2.

2. Rational blowdown

$$X = X \setminus C_p \cup C_p$$ plumbing

$$X' = X \setminus C_p \cup B_p$$ rational homology 4-ball



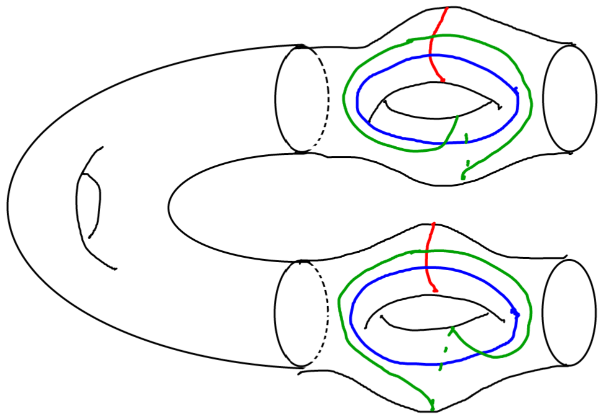
Trisection diagrams of C_p

Pieces of constructions of exotica

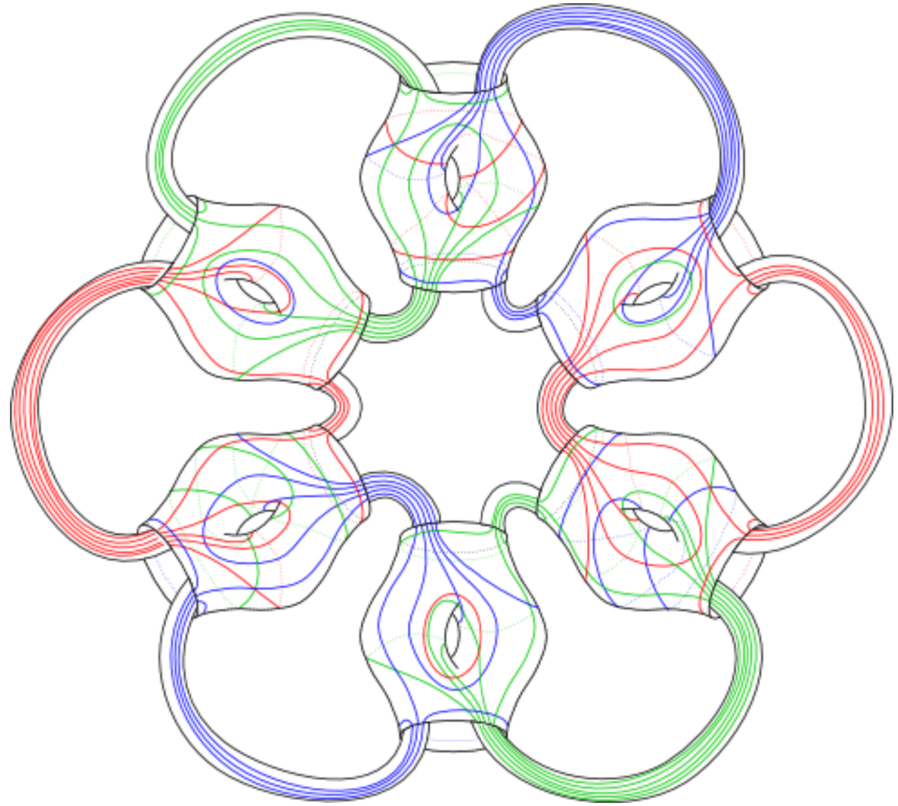
3. Fintushel-Stern knot surgery

$X = X \setminus N(T^2) \cup T^2 \times D^2$

$X' = X \setminus N(T^2) \cup S^1 \times S^3 \setminus N(K)$



Trisection diagram of $S^1 \times S^1 \times D^2$



Trisection diagram of $S^1 \times S^3 \setminus N(T_{2,3})$

Thank you!!!